

MOUNTAIN ORDERING: A METHOD FOR CLASSIFYING MOUNTAINS BASED ON THEIR MORPHOMETRY

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ABSTRACT

A new method for classifying mountain morphology, 'mountain ordering,' is proposed, and quantitative expressions for various morphological parameters of two ordered mountains in northern Japan were obtained using this method. Mountain order was defined in terms of the closed contour lines on a topographic map. A set of closed, concentric contour lines defines a first-order mountain. Higher-order mountains can be defined as a set of closed contour lines that contain lower-order mountains and that have only one closed contour line for each elevation; they are identified as $m + 1$ th-order mountains, where m represents the order of the enclosed, lower-order mountains. The geomorphometry for a mountain ordered according to this definition permits the identification of systematic relationships between various morphological parameters. The relationships between mountain order and these morphological parameters follow a form similar to that of Horton's laws, and permit the calculation of the ratios of number, area and height; these parameters are sufficient to express the magnitude of a mountain's dissection. The size–frequency distribution for area and height shows self-similarity for ordered mountains, and determines their fractal dimensions. Furthermore, the relationship between area and height, which has the form of a power function, describes the relief structure of ordered mountains. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: mountain ordering; morphology; morphometry; fractal analysis; relief structure

INTRODUCTION

The Earth's surface is shaped into various relief patterns, and any land with an elevation substantially higher than that of its surroundings is recognized as a mountain. Mountain morphology depends on geology or climate; and thus, the geomorphometry of mountains has been conducted in different areas using different methods (e.g. Strahler, 1952; Ahnert, 1970, 1984; Ohmori, 1978; Parsons and Abrahams, 1984; Schmidt and Montgomery, 1995). In particular, the application of fractal geometry and digital elevation models has led to a significant evolution in the techniques of mountain geomorphometry (e.g. Huang and Turcotte, 1989; Chase, 1992; Lifton and Chase, 1992; Ouchi and Matsushita, 1992).

These geomorphometrical studies have revealed the morphological characteristics of mountains and have discussed the development of mountains. The extent of a mountain, however, has not yet been determined by a certain definition, because of the lack of a geomorphological definition for mountains. Most morphological parameters (e.g. relief, slope and fractal dimension) vary in value as the selected extent of the mountain varies. Therefore, it is necessary to define the extent of a mountain before it is possible to determine the mountain's morphological parameters unequivocally.

This paper presents a method, 'mountain ordering,' for defining the extent of a mountain, and uses this method to examine the morphological characteristics of a set of ordered mountains in northern Japan. As an example of geomorphometry based on the method of mountain ordering, mountain order and the number, area and height for mountains of each order were determined for these mountains.

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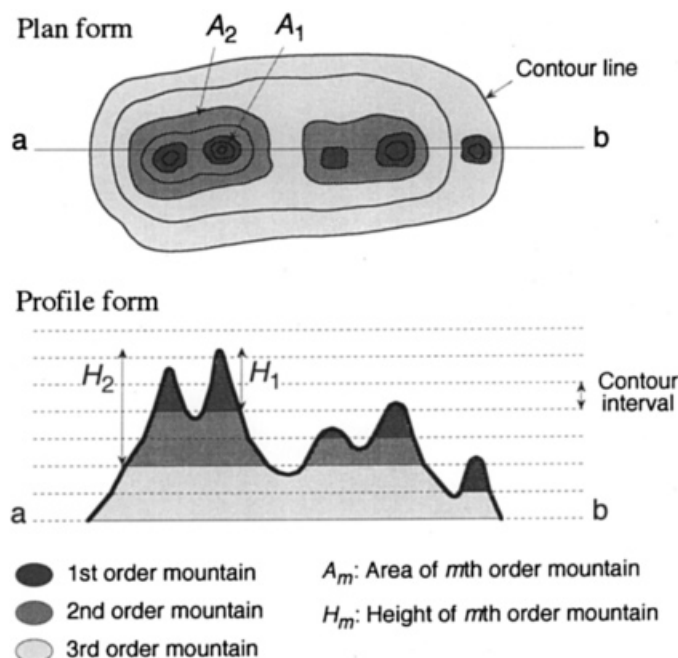


Figure 1. The definition of mountain order, and the areas and heights of ordered mountains. Although the second- and third-order mountains in this figure are only indicated by using different pixel patterns on the lower parts of the illustrations, these mountains nonetheless include the entire shaded upper parts. The area of an m th-order mountain, A_m , is defined as the inner area of the lowest-elevation contour line for that mountain, and was measured with a planimeter. The height of an m th-order mountain, H_m , is defined as $H_m = N_c I_c - 0.5 I_c$, where I_c is the height of a contour interval and N_c is the number of contour lines that make up the m th-order mountain

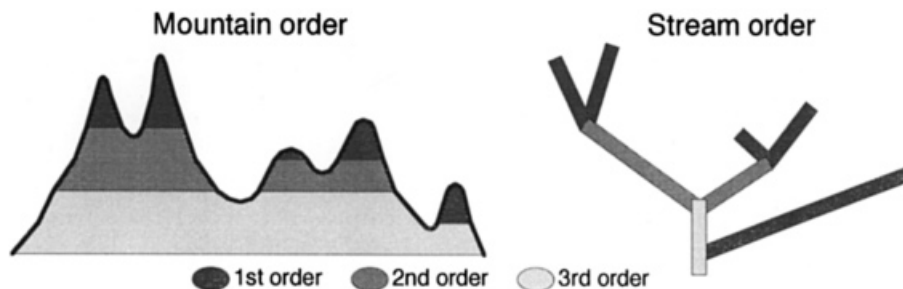


Figure 2. A comparison between the proposed mountain order system and the stream order system of Strahler (1952)

THE MOUNTAIN ORDERING METHOD

Mountain order is defined by the contour lines on a topographic map (Figure 1), in which each mountain is represented as a series of closed contour lines. These sets of contour lines include only a single closed contour line for each elevation unless a col that divides the mountain has a height that exceeds the contour interval. Thus, the closed contour lines form a series of concentric shapes. Each set of these contour lines defines a first-order mountain.

Higher-order mountains can be defined as a set of closed contour lines that contain the contour lines for one or more lower-order mountains and that have only one closed contour line for each elevation. If the highest of the enclosed lower-order mountains are of level m , then the surrounding higher-order mountain is identified as an $m + 1$ th-order mountain.

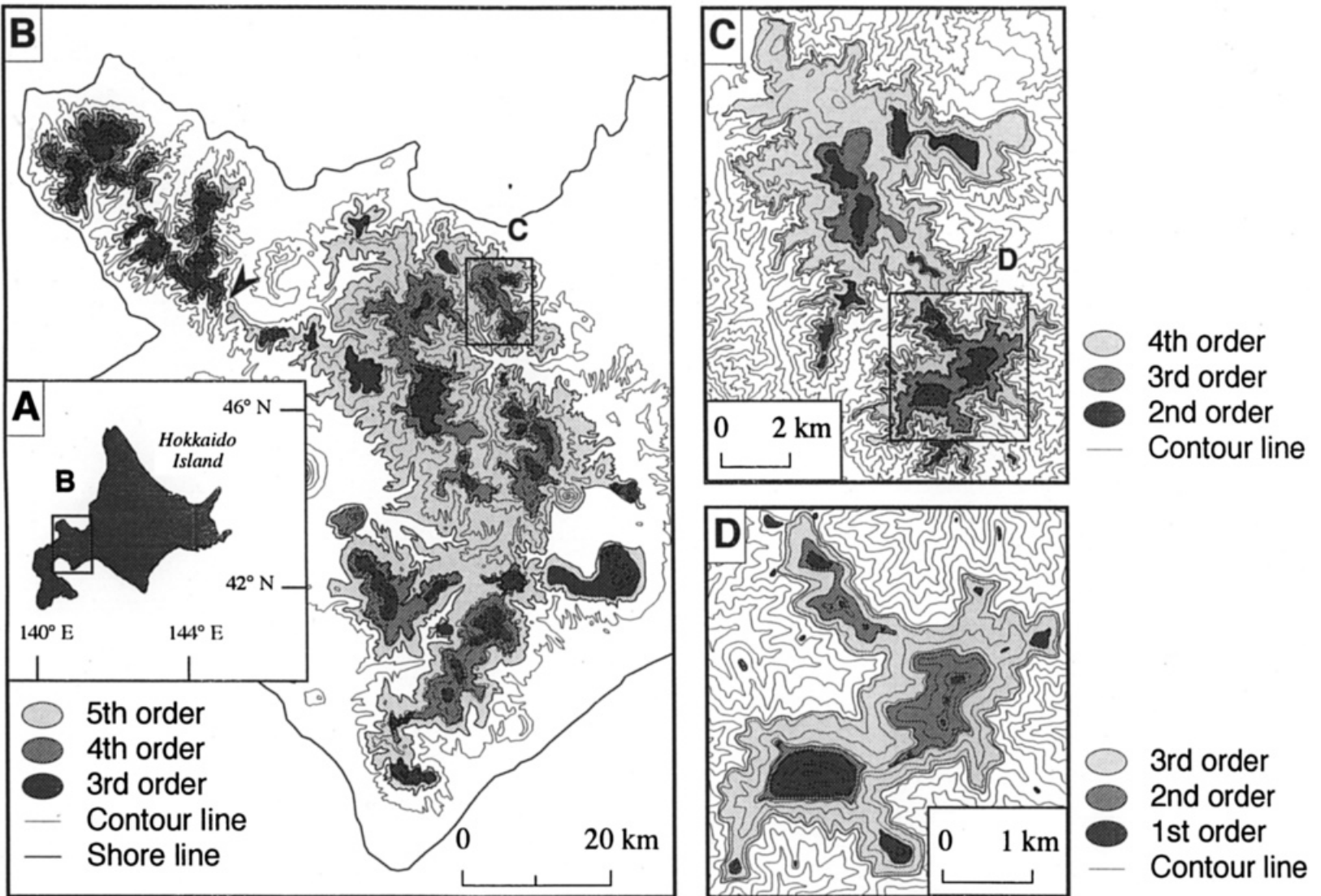


Figure 3. The study area (A) and the areal patterns of the ordered mountains (B, C and D). The contour intervals of maps B, C and D, are 200, 100 and 50 m, respectively. The arrow in B shows the col that divides the fifth-order mountains of the Northern and Southern parts. Although the first-order and second-order mountains are only illustrated in C or D, all the lower-order mountains contained within the fifth-order mountains illustrated in B were identified and measured during the study

Table 1. Morphological parameters of ordered mountains. A_g and H_g indicate geometrical mean of area and height, respectively

Order	Number, N	Area, A (km ²)			Height, H (km)		
		A_g	A_{\max}	A_{\min}	H_g	H_{\max}	H_{\min}
Northern part							
1st	541	0.008	0.66	0.001	0.011	0.185	0.005
2nd	68	0.18	2.3	0.01	0.059	0.255	0.015
3rd	10	6.7	24	1.2	0.272	0.525	0.155
4th	2	117	130	104	0.626	0.825	0.475
5th	1	369			0.895		
Southern part							
1st	2690	0.006	1.6	0.001	0.009	0.185	0.005
2nd	262	0.22	9.5	0.006	0.057	0.615	0.015
3rd	33	4.6	49	0.29	0.205	0.685	0.035
4th	6	73	154	22	0.454	0.705	0.205
5th	1	2251			1.085		

The proposed definition for mountain order is similar to that defined for stream order by Strahler (1952) (Figure 2). In accordance with Strahler's model, two first-order mountains can form a second-order mountain, but another first-order mountain at a lower elevation cannot make the second-order mountain into a third-order mountain; only the second-order mountain and another second-order mountain can form a third-order mountain.

The mountain order system implies that a well-dissected mountain range would be identified as a higher-order mountain, whereas a poorly dissected mountain such as a young stratovolcano would be identified as a lower-order mountain, if the selected areas of the two mountains were similar. A col that divides two higher-order mountains will be deeper than a col that divides the lower-order mountains that make up each higher-order mountain. Thus, a higher-order mountain would have a number of cols that vary widely in depth. Since a poorly dissected mountain would have a little col, it would be identified as a lower-order mountain unless the selected area expanded to encompass some additional mountains or mountain ranges.

MOUNTAIN ORDER IN A MOUNTAIN RANGE IN WESTERN HOKKAIDO

A mountain range in western Hokkaido (northern Japan) was selected to test the usefulness of the mountain ordering method in the geomorphometry of these mountains (Figure 3A). The length of the mountain range, which extended from 43° 20'N, 140° 30'E to 42° 20'N, 140° 90'E, was 120 km. The elevation of the highest point of the mountain range was 1480 m a.s.l. The rocks that form the mountains range from Tertiary and Quaternary andesite to Tertiary sedimentary rocks. A 1:25 000 topographic map (published by the Geographical Survey Institute of Japan) that used a contour interval of 10 m was used for the geomorphometry exercise.

Two fifth-order mountains were identified in the mountain range, which included approximately 3000 mountains ranging from first order to fourth order (Figure 3B, C and D). The fifth-order mountains were divided by a col at 400 m a.s.l. in the northwestern part of the mountain range. In this paper, the fifth-order mountain located on the northwestern side of that col is henceforth called the 'northern part'; the other fifth-order mountain is henceforth referred to as the 'southern part'.

The highest point in the southern part (1480 m a.s.l.) was slightly higher than that in the northern part (1290 m a.s.l.). The two parts differed significantly in area: the southern part (2251 km²) was more than six times the size of the northern part (369 km²).

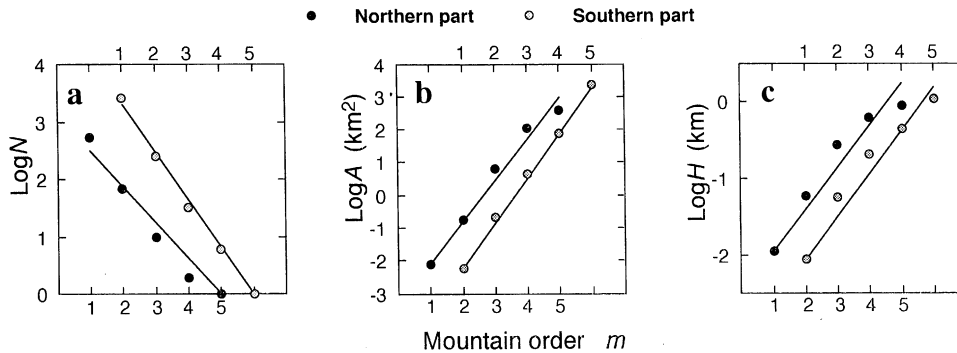


Figure 4. The number of mountains (N), their areas (A) and their heights (H) plotted against the corresponding mountain orders. A and H are the geometric mean values for the mountains in each mountain order. The equations for the best-fitting lines for the northern part are as follows: $N_m = 4.21j^{-m}$ ($r^2 = 0.96$), where j is the highest mountain order and m is the mountain order; $A_m = 0.008 \times 18.97^{m-1}$ ($r^2 = 0.97$); and $H_m = 0.011 \times 3.53^{m-1}$ ($r^2 = 0.91$). For the southern part, the best-fitting equations are as follows: $N_m = 6.70j^{-m}$ ($r^2 = 0.99$), $A_m = 0.006 \times 24.90^{m-1}$ ($r^2 = 0.99$) and $H_m = 0.009 \times 3.67^{m-1}$ ($r^2 = 0.95$)

MORPHOLOGICAL CHARACTERISTICS OF ORDERED MOUNTAINS

The morphological parameters of ordered mountains in the study area (number, N , area, A , and height, H), which are defined in Figure 1, were determined (Table I) and the data were analysed for each fifth-order mountain. The geometric means for each fifth-order mountain were used for the analysis of the areas and heights.

The number, area and height of the ordered mountains change systematically with changes in mountain order (Figure 4). The number of mountains of a given order approximates an inverse geometric series with respect to mountain order (Figure 4a). It follows that a mountain is divided into lower-order mountains at a constant rate, as is the case for streams (Horton, 1945), and the ratio, R_n , is given by:

$$N_m = R_n^{j-m} \quad (1)$$

where N_m is the number of m th-order mountains, and j is the highest order. The R_n values for the northern and southern parts of the study area were 4.21 and 6.70, respectively. A minimum of two m th-order mountains can form an $m + 1$ th order mountain; that is, the minimum R_n value is 2. The R_n values for the northern and southern parts were much larger than this minimum value. Since different heights of a col are needed to form a mountain with an order higher than 1, the number of m th-order mountains increases if the cols that divide the m th-order mountains are similar in height. This means that the R_n value becomes large for mountains that are dissected by cols with similar heights, but converges on 2 for mountains that are dissected by cols with different heights.

Area approximates a geometric series with respect to mountain order (Figure 4b). The area of a given mountain order, A_m , can be written as:

$$A_m = A_1 R_a^{m-1} \quad (2)$$

where A_1 is the geometric mean area for all first-order mountains, and R_a is the ratio of area. The R_a values for the northern and southern parts were 18.97 and 24.90, respectively; as was the case for R_n , the R_a value is larger for the southern part. Since the A_1 values for the two parts were similar, the area occupied by a given m th-order mountain for the northern part is smaller than the comparable value for the southern part. This result means that the mountains of the northern part, which have a lower R_a value, are more finely dissected than those of the southern part, which have a higher R_a value.

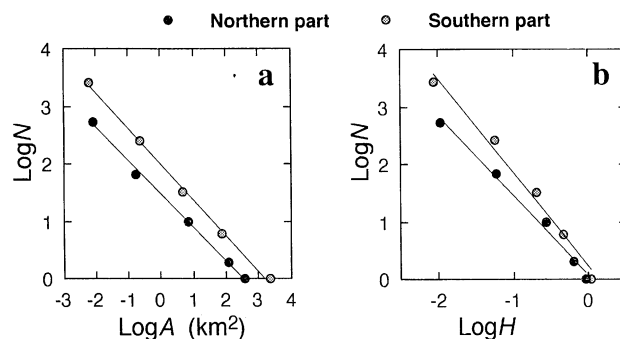


Figure 5. The area (A) and height (H) for the specific mountain orders plotted against the corresponding number of mountains of that order (N). A and H are the geometric mean values for the mountains in each order (m). The equations for the best-fitting lines for the northern part are as follows: $N_m = 29.88 A_m^{-0.58}$ ($r^2 = 0.99$) and $N_m = 1.1 H_m^{-1.41}$ ($r^2 = 0.99$). For the southern part, $N_m = 100.44 A_m^{-0.62}$ ($r^2 = 0.99$) and $N_m = 1.66 H_m^{-1.64}$ ($r^2 = 0.99$)

The heights of mountains of various orders also approximate a geometric series (Figure 4c). The height of a mountain of a given order, H_m , can be written as:

$$H_m = H_1 R_h^{m-1} \quad (3)$$

where H_1 is the geometric mean height for all first-order mountains, and R_h is the ratio of height. The R_h values for the northern and southern parts were 3.53 and 3.67, respectively; these values are similar. Larger R_h values show higher m th-order mountains if the H_1 values are similar. Because the H_1 values were similar for the northern and southern parts, the depths of the cols that dissect the two mountains are also similar.

The systematic relationships between mountain order and the selected morphological parameters suggest that the mountain system exhibits self-similarity, as is the case for streams (Horton, 1945). For ordered streams, the relationship between the number of tributaries and their lengths can be represented by a power function, and this shows that stream networks have fractal properties (Turcotte, 1997). The relationships between the number of mountains and their area can also be represented in the form of a power function (Figure 5a) as can those between the number of mountains and their height (Figure 5b), and this suggests that mountain orders also have fractal properties. This can be seen in the following two equations:

$$N_m = c_1 A_m^{-D_a/2} \quad (4)$$

$$N_m = c_2 H_m^{-D_h} \quad (5)$$

where m is a given mountain order, D_a and D_h are the fractal dimensions for area and height, respectively, and c_1 and c_2 are constants. These relationships demonstrate that the areal pattern and height distribution for ordered mountains are self-similar, and that the self-similarity is characterized by their respective fractal dimensions, D_a and D_h . The D_a values for the northern and southern parts were 1.16 and 1.24, respectively, and the corresponding D_h values were 1.41 and 1.64. Since the D_h values were larger than the D_a values, the area of ordered mountains increases faster than the height. This relationship is clearly shown in Figure 6, and can be defined by the expression:

$$H_m = r_c A_m^{\gamma/2} \quad (6)$$

where γ is the relief parameter and r_c is the characteristic relief. The γ values for the northern and southern parts were 0.80 and 0.74, respectively. Since both γ values are less than 1, the relief of ordered mountains (H /

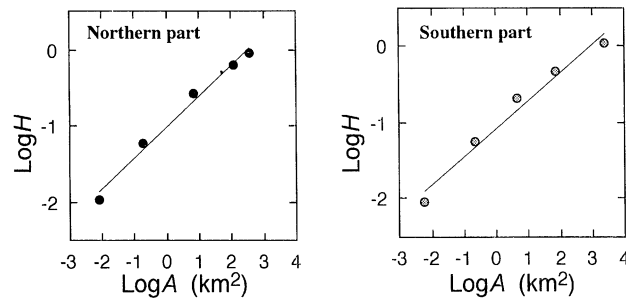


Figure 6. The relationship between the corresponding areas (A) and heights (H) for the specific mountain orders. A and H are the geometric mean values for the mountains of each order. The best-fitting equation for the northern part is $H_m = 0.097A_m^{0.40}$ ($r^2 = 0.99$); for the southern part, the best fit was obtained with the equation $H_m = 0.083A_m^{0.37}$ ($r^2 = 0.98$)

$A^{1/2}$) decreases with increasing mountain order. Therefore, for a mountain with $\gamma < 1$, the relief around the summit, which consists of lower-order mountains, is higher than that of the mountain from summit to piedmont, which consists of higher-order mountains. In addition to γ , r_c also shows the relief structure of an ordered mountain. Since r_c is the height per unit area, it represents a characteristic relief of the mountain. Thus, the relationship between A and H characterizes both the magnitude of the relief and its change from summit to piedmont.

CONCLUSIONS

The method of mountain ordering can define a mountain's extent, and yields quantitative expressions for describing the morphology of ordered mountains. The relationships between mountain order and various morphological parameters are similar in form to Horton's law and permit the calculation of the ratios for number, area and height; these parameters are sufficient to express the magnitude of the mountain's dissection. The size–frequency distributions for area and height suggest self-similarity for ordered mountains, and determine the fractal dimensions of the mountains. Furthermore, the relationship between area and height, which was found in this paper to take the form of a power function, describes the relief structure of the ordered mountains. Although further examination of the technique is necessary, the relationships obtained from the mountain ordering technique appear to describe adequately certain basic geomorphometric characteristics of a mountain.

Since the definition of a mountain's order is very simple, the morphological parameters presented in this paper and additional morphological parameters can be obtained from geomorphometry based on mountain ordering exercises conducted in various mountains over a broader range of areas. These morphological data will be a useful tool for discussing the development of mountains.

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